principle (which Green had found independently), the ever-widening range of applications, profound "pure" questions such as conditions for uniqueness of potentials and distributions and the existence of the attendant integrals, and so on. Some interesting historiographical questions arise: in particular, both Schwinger and Dyson stress the importance of Green's functions in the modern contexts, whereas in the classical period Green's theorem seems to have gained rather more attention. A third edition would be welcome, with a substantial new chapter reviewing this story, which led to Green becoming a scientific household name by the late 19th century.

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**Multigrid.** *By U. Trottenberg, C. Oosterlee, and A. Schüller.* Academic Press, London, 2001. $69.95. xv+631 pp., hardcover. ISBN 0-12-701070-X.

Multigrid is known to be the most efficient technique available for the numerical solution of large classes of partial differential equations (PDEs). Still, in practice, the use of the method is not as widespread as one might expect. One cause may be that there are few introductions to the subject that are accessible and at the same time give sufficient detail to see what difficulties can be encountered and how they can be overcome. The 10-year-old book *An Introduction to Multigrid Methods* by P. Wesseling is out of print. The new book *Multigrid* by U. Trottenberg, C. Oosterlee, and A. Schüller now fills the gap and gives an excellent treatment of the subject.

It is interesting to see how a new and unusual idea in mathematics, such as multigrid in scientific computing, develops. In the late 1970s the multigrid method had already been invented. The first papers by Fedorenko and Bakhvalov had appeared in Russia in the 1960s, but they didn't draw any attention.

In the 1970s A. Brandt recognized the importance of the idea and he advocated the technique in combination with adaptive grids, under the name MLAT (the multilevel adaptive technique), stressing that in all cases *the amount of computational work should be proportional to the real physical changes in the computed system.* He applied the method to some hard problems, ranging from semiconductor modeling to the Navier–Stokes equations. However, at that time much skepticism was felt about multigrid and the possible realization of the formulated goal.

Although nice results were obtained, the explanation of the success, based on Fourier analysis arguments by *local mode analysis*, was considered not sufficiently precise to give the method scientific prestige. In addition, the method is essentially recursive and, with its multiple grids, it requires a data structure that is more complex and consuming than those normally used for the solution of PDEs. This difficulty is probably still the major drawback of the method. As the computer language FORTRAN doesn't naturally support recursive routines and more complex data structures, for some time the use of this language hampered easy implementation. Nowadays a sufficient number of good codes are available, but the fact that the data structures needed for its application are relatively complex is still a disadvantage that makes the alternative Krylov-type methods more popular for linear problems.

Independent of the above development, starting in 1976, W. Hackbusch in Germany reinvented multigrid, providing it with a solid theoretical footing based on functional analysis tools familiar to numerical analysts in the framework of the finite element method. It was Hackbusch's book, in 1985, in which his earlier work was summarized, that made multigrid respectable in the eyes of those who thought that Brandt's aims were too ambitious. Now it was clear that multigrid allowed solution methods for PDEs in which the amount of computational work to solve a problem was indeed linearly proportional to the number of coefficients describing the solution (i.e., so-called $O(N)$-methods).

From this point on, multigrid research went in many directions. On the one hand, theorists were proving more and more abstract convergence theorems, especially for the linear symmetric positive definite case.

On the other hand, people struggled to realize the aim of computing solutions in $O(N)$ operations for complex technical problems. Nowadays the multigrid method is a well-respected technique that has found its place in scientific computing.

This book gives a comprehensive treatment, with an emphasis on the practical aspects of multigrid methods. The book is a most accessible introduction to elementary multigrid theory. The first 100 pages develop the basic ideas, and in the following 300 pages the book gradually develops toward real applications. Meanwhile the reader becomes acquainted with the techniques that are used to develop and understand the method in more complex situations.

Excellent examples of practical applications are given. Advanced techniques that are nowadays used in state-of-the-art systems, such as parallelization, adaptive and composite grids, or multigrid as a preconditioner, are also treated in detail.

Nevertheless, it is clear that the book is meant primarily for students and practitioners: engineers and people working in the computational sciences. Numerical analysts get their share in the end, and in particular in a few appendices that contain significant contributions by guest authors.

The first appendix is a 120-page "Introduction to Algebraic Multigrid" by K. Stüben, which explains how the multigrid idea can be applied for the solution of large linear systems without resorting to geometric meshes. It is an up-to-date and self-contained treatment of this important and increasingly popular subject.

The second appendix, by P. Oswald, is a brief (40 page) account of modern convergence theory, showing how convergence for multigrid and domain decomposition methods (etc.) can be studied in the framework of subspace correction methods.

The third contribution in the appendix, by A. Brandt, gives a list of possible difficulties a practitioner may encounter and advises about the corresponding solutions.

Altogether, I believe that *Multigrid* is a useful book in many respects. It gives a clear introduction to the student, it is a handbook for the practitioner, and for the expert it is a good reference and a nice compilation of knowledge that otherwise is only found scattered over the scientific literature. It is a book that should be on the desk of researchers in this area and in the library of any scientific computing department.

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**Limit Theorems of Probability Theory.** *Edited by Yu. V. Prokhorov and V. Statulevicius.* Springer-Verlag, Berlin, 2000. $104.00. x+273 pp., hardcover. ISBN 3-540-57045-4.

This book first appeared in a Russian language edition, published by VINITI, Moscow, 1991. It has a broad title, suggestive of comprehensive contents. However, the reality is rather limited, with five parts on different aspects of limit theory, some of a quite specialized nature. Nevertheless, it is useful for limit theory specialists to have a translation of this work readily available.

Part I, by V. V. Petrov, is concerned with the law of large numbers, the central limit theorem, and the law of the iterated logarithm for sums of independent random variables. It is in the nature of an introduction to the classical theory, and a more detailed presentation may be found elsewhere in book form, for example, in the author's book of 1987. Connections to the Brownian motion process and nonnormal limits are among the topics that are not included.

Part II, by V. Bentkus, F. Goetze, V. Paulauskas, and A. Rackauskas, deals with rates of convergence and asymptotic expansions associated with the central limit theorem for independent Banach-valued random elements. The account is in the nature of a review of methods, pointing out differences from the finite-dimensional case. Some applications to statistics are also provided through results related to the Cramér von Mises statistic, the $L$-statistic, Kolmogorov Smirnov statistics, and empirical processes.

Part III, by J. Sunklodas, principally surveys results on normal approximation of sums of weakly dependent random variables and random fields, the dependence being expressed in terms of mixing coef-